Quadrilaterals

A Diagonal of a Parallelogram Divides It into Two Congruent Triangles

The Diagonal of a Parallelogram Divides It into Two Congruent Triangles

Look at the two triangular sandwiches obtained by cutting a sandwich along the **diagonal**.

Are the two divided parts of the sandwich equal in **area**? What can we say about their congruency? Does the diagonal divide the sandwich into two triangles of equal area?

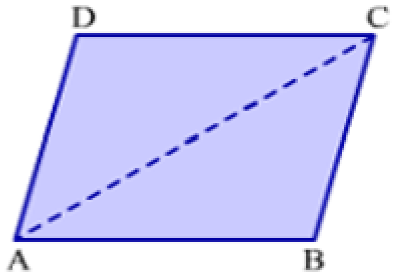
The answers to these questions are based on a property of **parallelograms**. It can be stated as follows:

A diagonal of a parallelogram divides it into two congruent triangles.

In this lesson, we will understand and prove the above-stated property of parallelograms. We will also solve some examples related to the same.

The Diagonal of a Parallelogram Divides It into Two Congruent Triangles

Consider a parallelogram ABCD. AC is a diagonal of this parallelogram.



Suppose the area of ΔABC is 15 cm^2.

We can find the area of the given parallelogram with the help of the area of ΔABC by using a property of parallelograms which states that:

A diagonal of a parallelogram divides it into two congruent triangles.

In the given figure, diagonal AC divides the parallelogram into two congruent triangles, ΔABC and ΔCDA.

We know that congruent figures are equal in area.

So, ar (ΔABC) = ar (ΔCDA)

∴ ar (parallelogram ABCD) = 2 × ar (ΔABC)

= 2 × 15 cm^2

= 30 cm^2

Concept Builder

- A trapezium is a quadrilateral having one pair of parallel opposite sides.
- A rectangle is a quadrilateral having two pairs of equal opposite sides. Also, each angle in a rectangle is equal to 90°.
- A rhombus is a quadrilateral having all sides equal.
- A square is a quadrilateral having all sides equal and all angles equal to 90°.

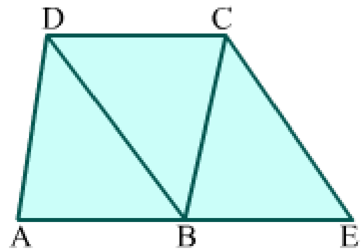
Did You Know?

- The quadrilateral formed by joining the midpoints of a quadrilateral is a parallelogram.
- If the diagonals of a quadrilateral are perpendicular to each other, then the quadrilateral formed by joining the midpoints of the quadrilateral is a rectangle.
- The sum of any three sides of a quadrilateral is always greater than the fourth side.
- The sum of all sides of a quadrilateral is greater than the sum of its diagonals.

Solved Examples

Easy

Example 1: **In the given figure, ABCD and BECD are two parallelograms. Prove that ΔABD ⍰ ΔBEC.**



Solution:

ABCD is a parallelogram with BD as a diagonal and BECD is a parallelogram with BC as a diagonal. We know that a diagonal of a parallelogram divides it into two congruent triangles.

∴ $\Delta ABD \cong \Delta CDB$

And $\Delta CDB \cong \Delta BEC$

$\Rightarrow \Delta ABD \cong \Delta BEC$

Example 2: **ABCD is a parallelogram with area 60 cm^2. Find the area of ΔACD.**

Solution:

We know that a diagonal of a parallelogram divides it into two congruent triangles.

Also, congruent figures are equal in area.

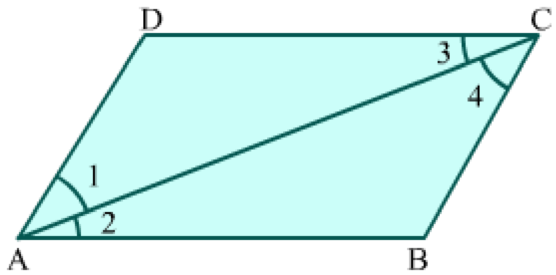
So, we have:

$$ar\,(\Delta ACD) = ar\,(\Delta CAB) = \frac{ar\,(\text{parallelogram } ABCD)}{2}$$

$$\Rightarrow ar\,(\Delta ACD) = \frac{60\,cm^2}{2}$$

$\Rightarrow \therefore ar\,(\Delta ACD) = 30\ cm^2$

Example 3: **ABCD is a parallelogram in which ∠1 = ∠2. Prove that ∠3 = ∠4.**



Solution:

ABCD is a parallelogram with AC as a diagonal. We know that a diagonal of a parallelogram divides it into two congruent triangles.

∴ $\Delta ABC \cong \Delta CDA$

⇒ $\angle 2 = \angle 3$ and $\angle 1 = \angle 4$ ... (1)

It is given that $\angle 1 = \angle 2$. ... (2)

From 1 and 2, we get:

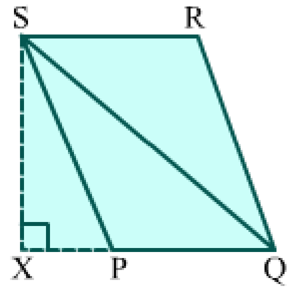
$\angle 3 = \angle 4$

Medium

Example 1: **The area of a parallelogram PQRS is 50 cm^2. Find the distance between PQ and SR if the length of PQ is 6 cm.**

Solution:

Construction: Draw diagonal SQ of the given parallelogram PQRS. Extend PQ and draw a line perpendicular to it from point S.



We know that a diagonal of a parallelogram divides it into two congruent triangles.

Also, congruent figures are equal in area.

∴ ar (ΔPQS) = ar (ΔRSQ)

Now, ar (parallelogram PQRS) = ar (ΔPQS) + ar (ΔRSQ)

⇒ ar (parallelogram PQRS) = 2 × ar (ΔPQS)

⇒ ar (ΔPQS) = $\frac{1}{2} \times$ ar (parallelogram PQRS)

⇒ ar (ΔPQS) = $\frac{50}{2}$ cm^2

⇒ ∴ ar (ΔPQS) = 25 cm^2

Also, ar (ΔPQS) = $\frac{1}{2} \times PQ \times SX$

⇒ $\frac{1}{2} \times PQ \times SX$ = 25 cm^2

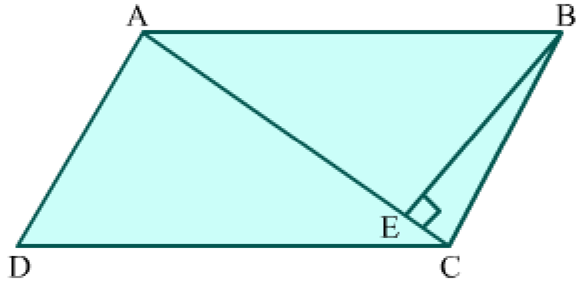
⇒ $PQ \times SX$ = 50 cm^2

⇒ SX = $\frac{50}{6}$ cm (∵ It is given that PQ = 6 cm)

⇒ ∴ SX = 8.3 cm

Thus, the distance between PQ and SR is 8.3 cm.

Example 2: In the given parallelogram ABCD, altitude BE on AC is of length 4 cm. If the length of diagonal AC is 6 cm, then find the area of the parallelogram.



Solution:

We know that a diagonal of a parallelogram divides it into two congruent triangles.

Also, congruent figures are equal in area.

∴ ar (ΔABC) = ar (ΔCDA)

Now, ar (ΔABC) = $\frac{1}{2}$ × Base × Height

$=\frac{1}{2}\times AC\times BE$

$=\frac{1}{2}\times 6\,cm\times 4\,cm$

$=12\,cm^2$

Therefore, ar (parallelogram ABCD) = ar (ΔABC) + ar (ΔCDA)

= 2 × ar (ΔABC)

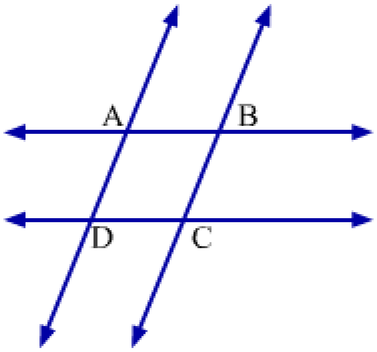
= 2 × 12 cm^2

= 24 cm^2

Properties of The Sides of a Parallelogram

Property of the Sides of a Parallelogram

Consider the given pairs of parallel lines.



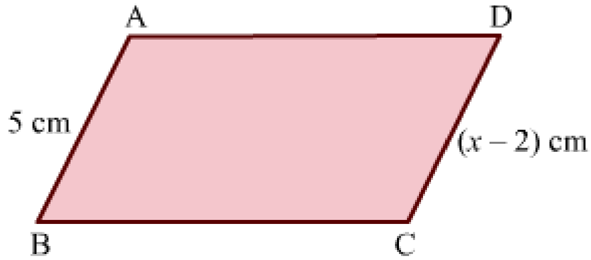
A closed figure ABCD is formed by the intersection of the two pairs of parallel lines. This figure is a parallelogram. A property of parallelograms defines the relation between the sides of a parallelogram as follows:

Opposite sides of a parallelogram are equal.

In this lesson, we will study the above-stated property and its converse. We will also solve some examples related to the same.

Opposite Sides of a Parallelogram Are Equal

Consider the given parallelogram ABCD.



We can find the value of x by using the property of parallelograms which states that:

Opposite sides of a parallelogram are equal.

Thus, in the given figure, we have AB = DC and AD = BC.

Since AB = DC, we have:

$x - 2 = 5$

$\Rightarrow x = 7$

Concept Builder

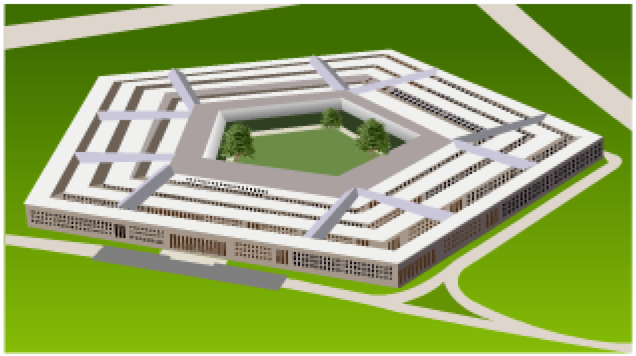
- A quadrilateral is a polygon having four sides.
- The sum of the interior angles of a quadrilateral is 360°.

Know More

- A pentadecagon is a fifteen-sided polygon. The sum of its interior angles is 2340°.
- An icosagon is a twenty-sided polygon. The sum of its interior angles is 3240°.

Did You Know?

The headquarters of the US Department of Defense is called ‘the Pentagon’. It is one of the world’s largest office buildings. It is virtually a city in itself.

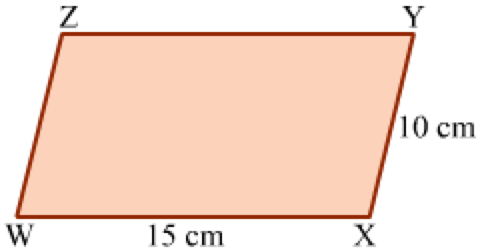


Solved Examples

Easy

Example 1: What is the perimeter of the given parallelogram WXYZ if WX = 15 cm and XY = 10 cm?

Solution:



We know that the opposite sides of a parallelogram are equal.

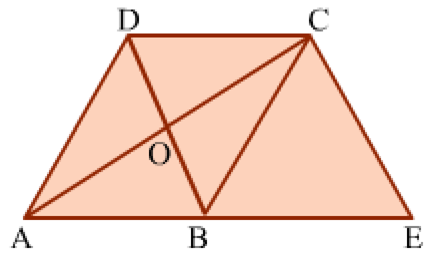
∴ WX = ZY = 15 cm and XY = WZ = 10 cm

Perimeter of parallelogram WXYZ = WX + XY + YZ + ZW
= (15 + 10 + 15 + 10) cm

= 50 cm

Medium

Example 1: In the given figure, ABCD is a parallelogram and B is the midpoint of AE. If DB = CE, then prove that BECD is also a parallelogram.



Solution:

We know that the opposite sides of a parallelogram are equal.

∴ AB = DC … (1)

It is given that B is the midpoint of AE.

∴ AB = BE … (2)

From equations 1 and 2, we get:

DC = BE

Also, it is given that DB = CE.

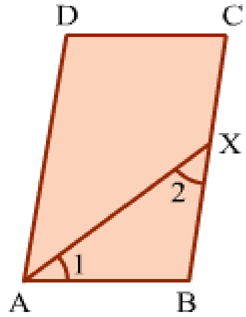
Now, in quadrilateral BECD, the opposite sides are equal (i.e., DC = BE and DB = CE). Therefore, it is a parallelogram.

Hard

Example 1: In a parallelogram ABCD, the bisector of ∠BAD also bisects side BC. Prove that the length of side AD is twice the length of side AB.

Solution:

The parallelogram ABCD according to the given specifications is shown below.



Here, AX is the bisector of ∠BAD.

$\therefore \angle 1 = \frac{1}{2}\angle BAD$ ...(1)

Since ABCD is a parallelogram, AD||BC and AB is the transversal between these lines.

∴ ∠BAD + ∠CBA = 180° ... (2)

In ΔABX, by the angle sum property of triangles, we have:

∠1 + ∠2 + ∠ABX = 180°

$\Rightarrow \frac{1}{2}\angle BAD + \angle 2 + 180° - \angle BAD = 180°$ (Using equations 1 and 2)

$\Rightarrow \angle 2 + \frac{1}{2}\angle BAD = 0$

$\Rightarrow \angle 2 = \frac{1}{2}\angle BAD$

$\Rightarrow \angle 2 = \angle 1$

We know that the sides opposite equal angles are also equal.

∴ AB = BX ... (3)

Since ABCD is a parallelogram, AD = BC.

Now, BC = BX + XC

⇒ AD = BX + XC

⇒ AD = 2BX(∵ AX bisects BC)

⇒ ∴ AD = 2AB (Using equation 3)

Thus, in parallelogram ABCD, the length of side AD is twice the length of side AB.

Properties of The Angles of a Parallelogram

Look at the postage stamp shown below.

Observe how the stamp is shaped like a parallelogram. What can you say about its opposite angles? Is there any relation between them? Are they equal?

A property of parallelograms relates the opposite angles of a parallelogram as follows:

Opposite angles of a parallelogram are equal.

In this lesson, we will study the above-stated property and its converse. We will also solve some examples based on the same.

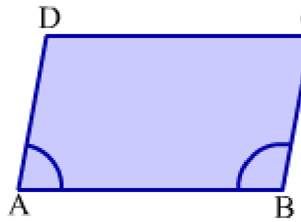
Whiz Kid

The sum of the measures of all the exterior angles of a quadrilateral (i.e., one at each vertex) is equal to the sum of the measures of all the interior angles of the quadrilateral, i.e., 360°.

Concept Builder

Adjacent angles in a parallelogram are supplementary.

In parallelogram ABCD, AD||BC and AB is the transversal intersecting these lines.



Therefore, ∠A and ∠B are **interior angles**

on the same side of the transversal and, hence, **supplementary**

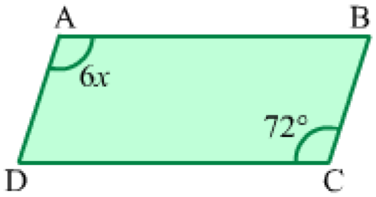
.

Similarly, we can say that ∠B and ∠C, ∠C and ∠D, and ∠D and ∠A are supplementary angles.

Solved Examples

Easy

Example 1: **Find the value of *x* if ABCD is a parallelogram.**



Solution:

We know that the opposite angles of a parallelogram are equal.

∴ ∠A = ∠C

⇒ $6x = 72°$

∴ $x = 12°$

Example 2: Find the measure of all the angles of a parallelogram whose adjacent angles are in the ratio 1:2.

Solution:

In a parallelogram ABCD, let $\angle A = x°$ and $\angle B = 2x°$.

In a parallelogram, the adjacent angles are supplementary.

$\angle A + \angle B = 180°$

$\Rightarrow x° + 2x° = 180°$

$\Rightarrow 3x° = 180$

$\Rightarrow x° = \frac{180°}{3}$

$\Rightarrow x° = 60°$

Thus, we get

$\angle A = x°$ = 60°

$\angle B = 2x°$ = 2 × 60° = 120°

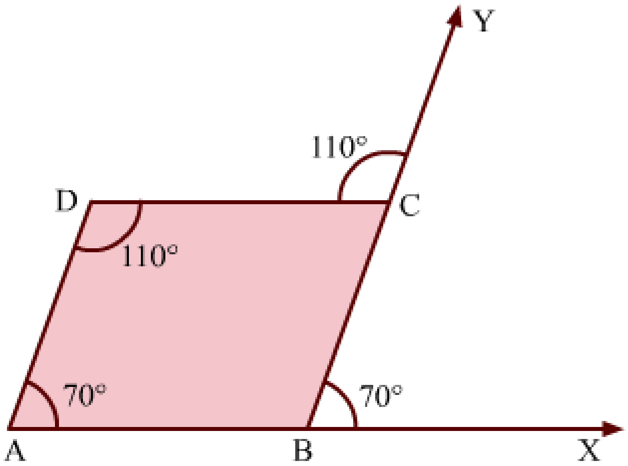
In a parallelogram, the opposite angles are equal.

Thus, we get

∠A = ∠C = 60° and ∠B = ∠D = 120°

Medium

Example 1: **Is the shown quadrilateral ABCD a parallelogram?**



Solution:

In the given figure, ∠CBX and ∠CBA form a linear pair.

∴ ∠CBX + ∠CBA = 180°

⇒ ∠CBA = 180° – ∠CBX

⇒ ∠CBA = 180° – 70°

⇒ ∠CBA = 110°

⇒ ∠CBA = ∠CDA

Similarly, ∠DCY and ∠BCD form a linear pair.

∴ ∠DCY + ∠BCD = 180°

⇒ ∠BCD = 180° – ∠DCY

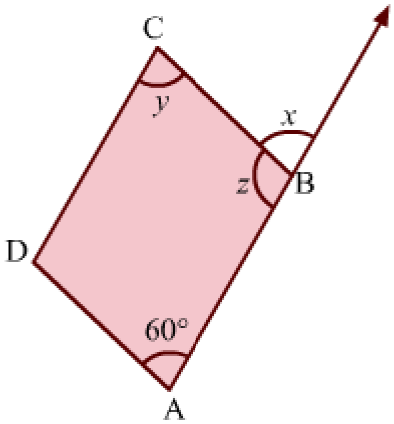
⇒ ∠BCD = 180° – 110°

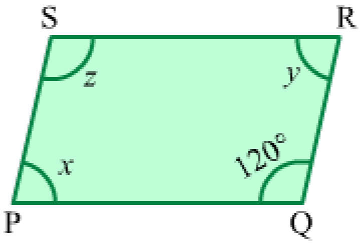
⇒ ∠BCD = 70°

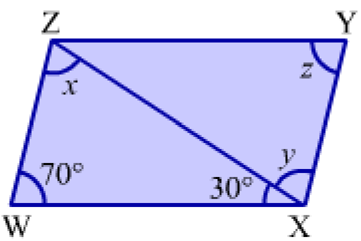
⇒ ∠BCD = ∠BAD

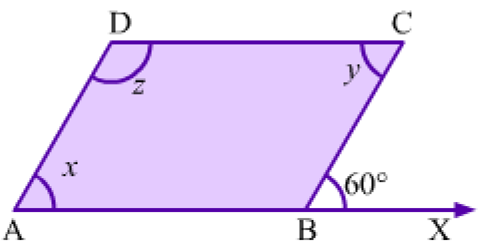
Thus, quadrilateral ABCD has two pairs of equal opposite angles. Hence, it is a parallelogram.

Example 2: **Find the values of *x*, *y* and *z* in the following parallelograms.**

1. 

2. 

3. 

4. 

Solution:

1. We know that the opposite angles of a parallelogram are equal.

   So, ∠BCD = ∠DAB

   ∴ y = 60°

We also know that the adjacent angles of a parallelogram are supplementary.

So, ∠CBA + ∠DAB = 180°

⇒ $z + 60° = 180°$

⇒ $z = 180° - 60°$

⇒ ∴ $z = 120°$

Now, x and z form a linear pair of angles; so, their sum is 180°.

So, $x + z = 180°$

⇒ $x + 120° = 180°$

⇒ ∴ $x = 180° - 120° = 60°$

2. We know that the opposite angles of a parallelogram are equal.

So, ∠PSR = ∠PQR

∴ $z = 120°$

∠QPS and ∠PQR are adjacent angles.

So, ∠QPS + ∠PQR = 180°

□ $x + 120° = 180°$

□ $x = 180° - 120°$

□ □ $x = 60°$

∠QRS and ∠QPS are opposite angles.

So, ∠QRS = ∠QPS

□ $y = x$

□ ∴ $y = 60°$

3. We know that the opposite angles of a parallelogram are equal.

So, ∠XYZ = ∠XWZ

 $z = 70°$

∠XYZ and ∠WXY are adjacent angles.

 ∠XYZ + ∠WXY = 180°

 $z + 30° + y = 180°$

 $70° + 30° + y = 180°$

 $100° + y = 180°$

 $y = 180° - 100°$

 $\therefore y = 80°$

Now, XY||WZ; so, ∠WZX and ∠YXZ are alternate interior angles.

So, ∠WZX = ∠YXZ

 $x = y$

$\therefore x = 80°$

4. It is given that ∠CBX = 60°.

   ∠CBA and ∠CBX form a linear pair.

   So, ∠CBA + ∠CBX = 180°

    ∠CBA + 60° = 180°

    ∠CBA = 180° − 60°

    ∴ ∠CBA = 120°

   ∠CDA and ∠CBA are opposite angles.

   So, ∠CDA = ∠CBA

    z = ∠CBA

⇒ ⇒ $z = 120°$

∠BCD and ∠CBA are adjacent angles.

So, ∠BCD + ∠CBA = 180°

⇒ $y + 120° = 180°$

⇒ $y = 180° - 120°$

⇒ ∴ $y = 60°$

∠BAD and ∠BCD are opposite angles.

So, ∠BAD = ∠BCD

⇒ $x = y$

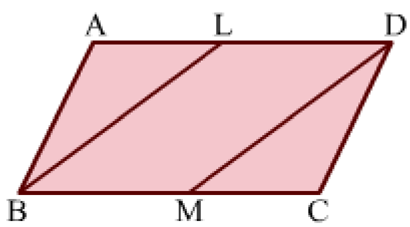
⇒ ⇒ $x = 60°$

Hard

Example 1: **Show that the bisectors of opposite angles of a parallelogram are parallel to each other.**

Solution:

Let ABCD be a parallelogram. Let BL and DM be the bisectors of ∠ABC and ∠ADC respectively.



Since BL and DM are the bisectors of ∠ABC and ∠ADC respectively, we have:

$$\angle LBM = \frac{\angle ABC}{2} \qquad ...(1)$$

$$\angle LDM = \frac{\angle ADC}{2} \qquad ...(2)$$

We know that the opposite angles of a parallelogram are equal.

∴ ∠ABC = ∠ADC

On dividing both sides of the above equation by 2, we obtain:

$$\frac{\angle ABC}{2} = \frac{\angle ADC}{2}$$

Using equations 1 and 2, we obtain:

∠LBM = ∠LDM

Now, LD and BM are parallel.

So, ∠DLB + ∠LBM = 180° (Interior angles on the same side of a transversal)

⇒ ∠DLB = 180° − ∠LBM

Similarly, ∠DMB = 180° − ∠LDM

∴ ∠DLB = ∠DMB (∵ ∠LBM = ∠LDM)

In quadrilateral LDMB, the opposite angles ∠DLB and ∠DMB are equal. Hence, it is a parallelogram.

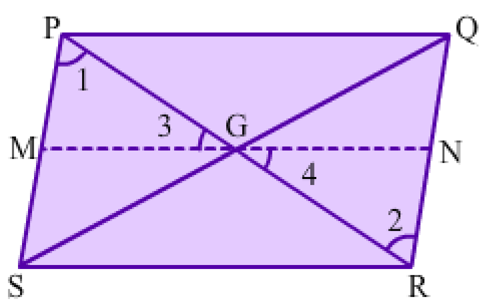
⇒ BL||DM

We know that BL and DM are the bisectors of opposite angles of parallelogram ABCD. Thus, the bisectors of opposite angles of a parallelogram are parallel.

Properties of The Diagonals of a Parallelogram

Relation between the Diagonals of a Parallelogram

Consider the following parallelogram PQRS.



In the figure, GM = GN, but can we prove this?

In order to prove GM = GN, we need to show that ΔGMP is congruent to ΔGNR.

In ΔGMP and ΔGNR, we have two sets of equal angles as follows:

∠3 = ∠4(Vertically opposite angles)

∠1 = ∠2(Alternate interior angles; since PS||QR and PR is the transversal)

Now, to apply the ASA congruence rule, we need to show that GP and GR are equal.

A property of parallelograms helps us establish this equality and it can be stated as follows:

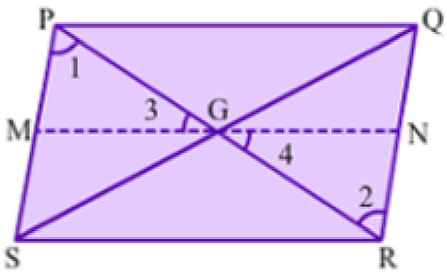
The diagonals of a parallelogram bisect each other.

In this lesson, we will study the above-stated property and solve some problems based on it.

Using the Property

Let us use the property of the diagonals of a parallelogram to solve the problem discussed at the beginning.

Let us once again consider parallelogram PQRS.



We have to prove that GM = GN.

Since diagonals PR and QS bisect each other, we obtain:

GP = GR and GS = GQ… (1)

In ΔGMP and ΔGNR, we have:

∠3 = ∠4(Vertically opposite angles)

∠1 = ∠2(Alternate interior angles; since PS||QR and PR is the transversal)

GP = GR(Using 1)

Thus, by the ASA congruence rule, we obtain:

ΔGMP ≅ ΔGNR

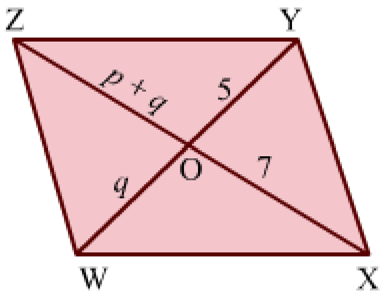
⇒ GM = GN(By CPCT)

Similarly, we can use the property of the diagonals of a parallelogram to solve other problems.

Solved Examples

Easy

Example 1: If the shown quadrilateral WXYZ is a parallelogram, then find the values of *p* and *q*.



Solution:

We know that the diagonals of a parallelogram bisect each other.

∴ WO = OY

⇒ $q = 5$

Similarly, XO = OZ

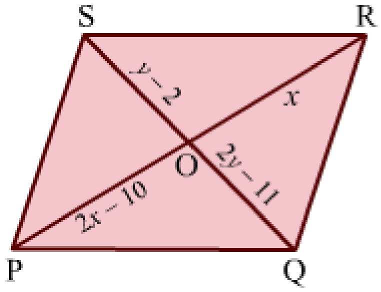
∴ $p + q = 7$

⇒ $p = 7 - q$

⇒ $p = 7 - 5$

⇒ ∴ $p = 2$

Example 2: In the given parallelogram PQRS, find the lengths of the diagonals PR and QS.



Solution:

We know that the diagonals of a parallelogram bisect each other.

∴ PO = OR

⇒ $2x - 10 = x$

⇒ $2x - x = 10$

⇒ ∴ $x = 10$

Similarly, QO = OS

⇒ $y - 2 = 2y - 11$

⇒ $2y - y = -2 + 11$

⇒ ∴ $y = 9$

Now, PR = PO + OR

= $2x - 10 + x$

= $3x - 10$

= $3 \times 10 - 10$

= $30 - 10$

= 20

Similarly, QS = QO + OS

= y − 2 + 2y − 11

= 3y − 13

= 3 × 9 − 13

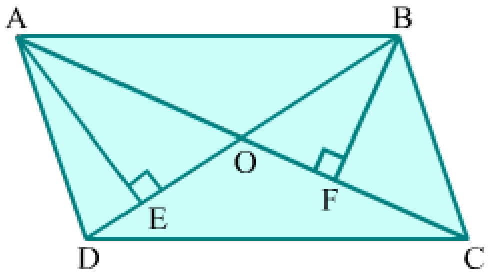
= 27 − 13

= 14

Thus, the lengths of the diagonals PR and QS are 20 units and 14 units respectively.

Medium

Example 1: ABCD is a parallelogram with diagonals AC and BD of lengths 10 cm and 8 cm respectively. If the perpendiculars on DO and OC are 5 cm each, then find the sum of the areas of ΔAOD and ΔBOC.



Solution:

We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, we have:

AO = OC = $\frac{AC}{2}$ and BO = OD = $\frac{BD}{2}$

⇒ AO = OC = $\frac{10}{2}$ cm = 5 cm and BO = OD = $\frac{8}{2}$ cm = 4 cm

Now, area of ΔAOD= $\frac{1}{2}$ × Base × Height

$= \frac{1}{2} \times OD \times AE$

$= \frac{1}{2} \times 4 \times 5\ cm^2$

$= 10\ cm^2$

Similarly, area of $\Delta BOC = \frac{1}{2} \times OC \times BF$

$= \frac{1}{2} \times 5 \times 5\ cm^2$

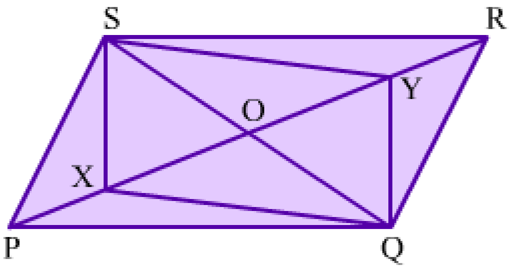
$= 12.5\ cm^2$

Therefore, sum of the areas of ΔAOD and ΔBOC = $(10 + 12.5)\ cm^2 = 22.5\ cm^2$

Hard

Example 1: In parallelogram PQRS, X and Y are points on PR such that PX = YR. Prove that:

1. **XQYS is a parallelogram**
2. **$\Delta SXP \cong \Delta QYR$**



Solution:

1. We know that the diagonals of a parallelogram bisect each other.

   ∴ OS = OQ … (1)

   And OP = OR… (2)

   Also, PX = YR… (3) [Given]

On subtracting equation 3 from equation 2, we obtain:

OP – PX = OR – YR

⇒ OX = OY ... (4)

In quadrilateral XQYS, XY and QS are the diagonals.

We know from equations 1 and 4 that the diagonals bisect each other.

Thus, XQYS is a parallelogram.

2. In ΔSXP and ΔQYR, we have:

   PS = QR (Opposite sides of parallelogram PQRS)

   SX = QY (Opposite sides of parallelogram XQYS)

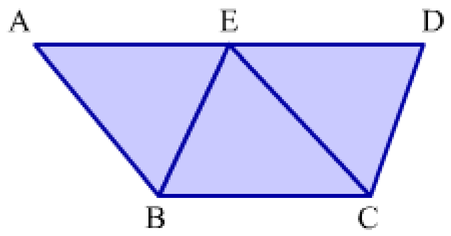
   PX = YR (Given)

   ∴ ΔSXP ≅ ΔQYR (By the SSS congruence rule)

A Quadrilateral is a Parallelogram if a Pair of Opposite Sides is Equal and Parallel

Necessary Condition for a Quadrilateral to Be a Parallelogram

Consider the given figure.



Here, ABCE is a parallelogram and AE is extended to D such that AE = ED.

Using an important property of parallelograms, we can prove that BCDE is also a parallelogram.

The property used for proving the above can be stated as follows:

A quadrilateral is a parallelogram if it has one pair of parallel and equal (or congruent) opposite sides.

Since ABCE is a parallelogram, AE||BC and AE = BC.

Also, AE = ED ⇒ ED = BC

And AE||BC ⇒ AD||BC ⇒ ED||BC

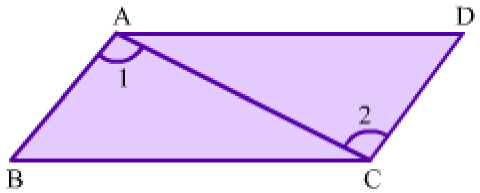
Hence, by the above-stated property, BCDE is a parallelogram.

In this lesson, we will understand and prove this property of parallelograms. We will also solve some examples based on the same.

Solved Examples

Easy

Example 1: **In the given figure, ⍰1 = ⍰2 and AB = DC. Is quadrilateral ABCD a parallelogram?**



Solution:

In quadrilateral ABCD, ∠1 = ∠2(Given)

These angles are alternate angles made by the transversal AC between lines AB and DC.

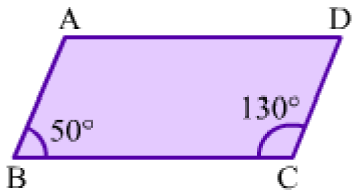
We know that if equal alternate angles are made by a transversal between two lines, then the lines intersected by the transversal are parallel.

∴ AB||DC

Also, AB = DC (Given)

We know that a quadrilateral is a parallelogram if it has a pair of parallel and equal opposite sides. Thus, ABCD is a parallelogram.

Example 2: **In the given figure, AB = DC. Prove that AD||BC.**



Solution:

In quadrilateral ABCD, ∠B + ∠C = 50° + 130° = 180°

We know that if the interior angles on the same side of a transversal are supplementary, then the lines intersected by the transversal are parallel.

⇒ AB||DC

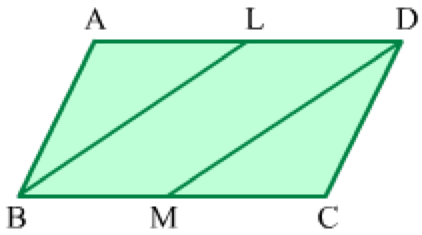
Also, AB = DC (Given)

We know that a quadrilateral is a parallelogram if it has a pair of parallel and equal opposite sides. Thus, ABCD is a parallelogram.

⇒ AD||BC (∵ Opposite sides of a parallelogram are parallel)

Medium

Example 1: **ABCD is a parallelogram in which L and M are the midpoints of AD and BC respectively. Prove that BMDL is a parallelogram.**



Solution:

It is given that L and M are the midpoints of AD and BC respectively.

$\therefore BM = \frac{1}{2}BC \text{ and } LD = \frac{1}{2}AD$

ABCD is a parallelogram. (Given)

So, BC = AD (∵ Opposite sides of a parallelogram are equal)

$\therefore \frac{1}{2}BC = \frac{1}{2}AD$

⇒ BM = LD ... (1)

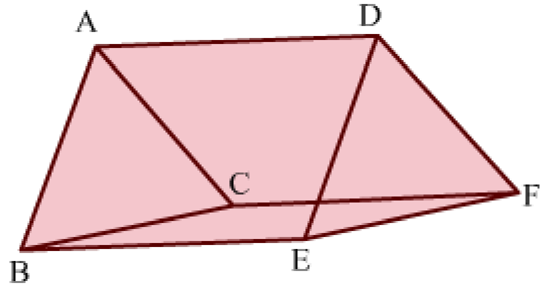
Also, BC||AD (∵ Opposite sides of a parallelogram are parallel)

⇒ BM||LD ... (2)

We know that a quadrilateral is a parallelogram if it has a pair of parallel and equal opposite sides. From 1 and 2, we conclude that BMDL is a parallelogram.

Hard

Example 1: **Sides AB and BC of ΔABC are parallel and equal to the corresponding sides DE and EF of ΔDEF. Prove that ACFD is a parallelogram.**



Solution:

Consider quadrilateral ABED.

We have AB = DE and AB||DE (Given)

Hence, ABED is a parallelogram. (∵ There is one pair of equal and parallel opposite sides)

⇒ AD = BE and AD||BE... (1)

Now, consider quadrilateral BCFE.

We have BC = EF and BC||EF (Given)

Hence, BCFE is a parallelogram.(∵ There is one pair of equal and parallel opposite sides) ⇒ CF = BE and CF||BE... (2)

From 1 and 2, we have:

AD = CF and AD||CF

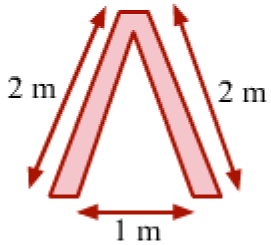
Hence, ACFD is a parallelogram. (∵ There is one pair of equal and parallel opposite sides)

Mid Point Theorem and Its Converse

Midpoint Theorem

The given image is that of a food outlet named Adam's Diner. The logo of the outlet, as you can see on top of the building, is the letter 'A'.

The carpenter who made the logo fixed the two legs of the logo, each of length 2 m, such that the distance between the two legs at the base was 1 m.

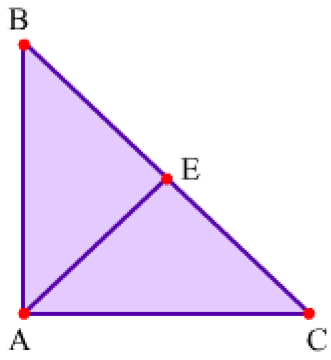


The carpenter then put the horizontal bar of the logo at the **mid-point** of the two legs so that it fitted exactly in the middle of the legs. The midpoint theorem came to his aid while doing so. In this lesson, we will study this theorem, its proof and its converse, and solve some related examples.

Whiz Kid

Right triangle median theorem

The length of the median on the hypotenuse of a right triangle is half that of the hypotenuse.



Here, AE = $\frac{1}{2}$ × BC

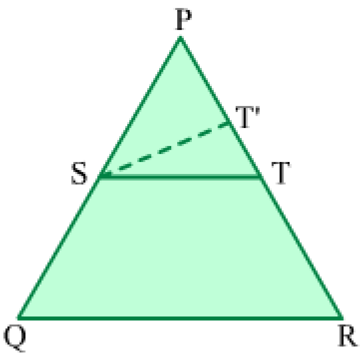
Activity

Perform the following activity to verify the midpoint theorem.

- Draw a large scalene triangle on a sheet of paper.
- Name its vertices A, B and C. Find the midpoints of sides AB and AC and name them D and E respectively. Draw a line joining the two midpoints.
- Cut out ΔABC and then cut it along line segment DE.
- Draw a quadrilateral BDEC. Place ΔADE on it such that vertices E and D of the triangle are respectively on vertex C and line segment BC of the quadrilateral. Mark the point where vertex D touches line segment BC.
- Shift ΔADE such that its vertices D and E are respectively on vertex B and line segment BC of the quadrilateral. Mark the point where vertex E touches line segment BC.
- What do you notice about the lengths of DE and BC? Do the marked points coincide?

Proof of the Converse of the Midpoint Theorem

Statement:The line drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side.



Given: ΔPQR in which S is the midpoint of PQ and ST||QR

To prove: T is the midpoint of PR

Construction: Take a point T′ on PR and join S and T′.

Proof: Let us say that T is not the midpoint of PR. Let T′ be the midpoint of PR.

In ΔPQR, S is the midpoint of PQ and T′ is the midpoint of PR.

∴ By the midpoint theorem, ST′||QR … (1)

It is given that ST||QR. … (2)

From 1 and 2, we conclude that ST||ST′, which is not possible.

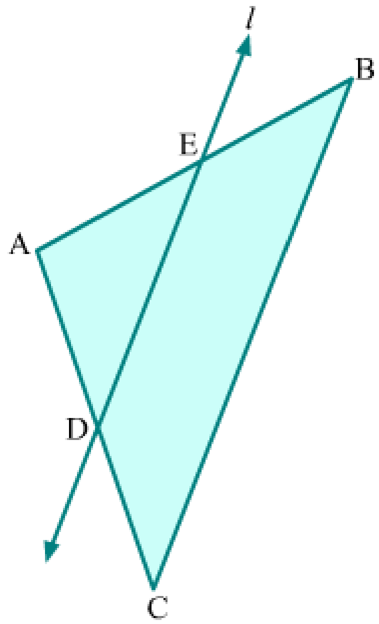
So, our assumption was wrong.

Hence, T is the midpoint of PR.

Solved Examples

Easy

Example 1: **Line *l* cuts the isosceles ΔABC, with AB = AC = 10 cm and BC = 15 cm, in such a way that ΔADE is also isosceles with AD = AE = 5 cm. Find the length of DE.**



Solution:

It is given that ΔABC is isosceles with AB = AC = 10 cm, BC = 15 cm and AD = AE = 5 cm.

AC = AD + DC

⇒ DC = AC − AD = (10 − 5) cm = 5 cm

Since AD = DC, D is the midpoint of AC.

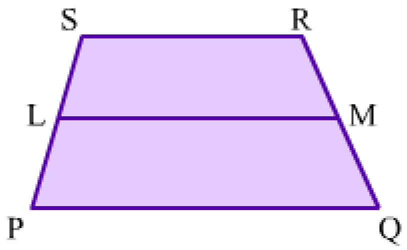
Similarly, E is the midpoint of AB.

Using the midpoint theorem, we get:

DE = $\frac{BC}{2} = \frac{15}{2}$ cm = 7.5 cm

Medium

Example 1: **In the given trapezium PQRS, PQ||SR and L is the midpoint of line segment PS. If LM||PQ, then prove that RM = QM.**



Solution:

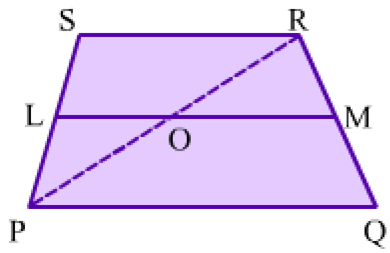
It is given that:

LM||PQ

PQ||SR

⇒ LM||SR(∵ Lines parallel to the same line are also parallel)

Construction: Join P to R such that PR intersects LM at point O.



In ΔPSR, L is the midpoint of PS and LO||RS.

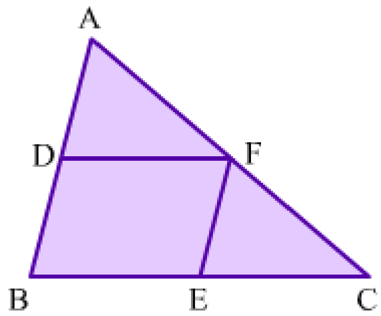
So, by the converse of the midpoint theorem, we get O as the midpoint of PR.

Now, in ΔPQR, O is the midpoint of PR and OM||PQ.

Again, by the converse of the midpoint theorem, we get M as the midpoint of QR.

⇒ RM = QM

Example 2: In the shown ΔABC, D, E and F are the midpoints of sides AB, BC and AC respectively. Prove that BEFD is a parallelogram.



Solution:

In ΔABC, D and F are the midpoints of AB and AC respectively.

So, by the midpoint theorem, we obtain DF||BC

⇒ DF||BE ... (1)

Again, in ΔABC, E and F are the midpoints of BC and AC respectively.

So, by the midpoint theorem, we obtain EF||BA

⇒ EF||BD ... (2)

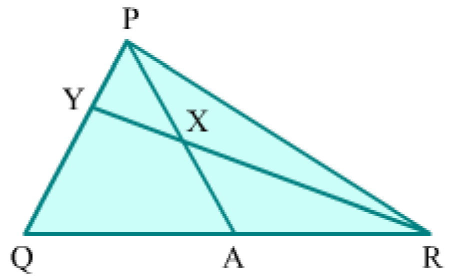
Now in quadrilateral BEFD, we have:

DF||BE and EF||BD (From 1 and 2)

⇒ BEFD is a parallelogram. (∵ There are two pairs of parallel opposite sides)

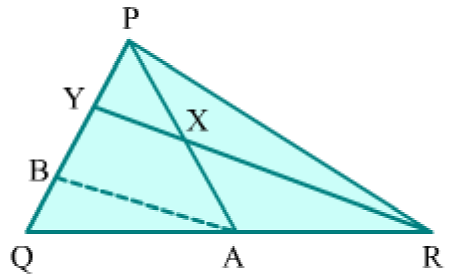
Hard

Example 1: In the given ΔPQR, PA is the median to side QR and X is the midpoint of PA. When produced, RX meets PQ at Y. Prove that $PY = \frac{1}{3}PQ$.



Solution:

Construction: Draw a line AB such that it is parallel to RY.



In ΔPAB, X is the midpoint of AP and XY||AB.

So, by the converse of the midpoint theorem, we obtain Y as the midpoint of PB.

⇒ PY = YB … (1)

In ΔQYR, A is the midpoint of QR. (∵ AP is the median to side QR)

Also, RY||AB

So, by the converse of the midpoint theorem, we obtain B as the midpoint of YQ.

$\Rightarrow$ YB = BQ ... (2)

From equations 1 and 2, we obtain:

PY = YB = BQ ... (3)

From the figure, we have:

PQ = PY + YB + BQ

$\Rightarrow$ PQ = PY + PY + PY

$\Rightarrow$ PQ = 3PY

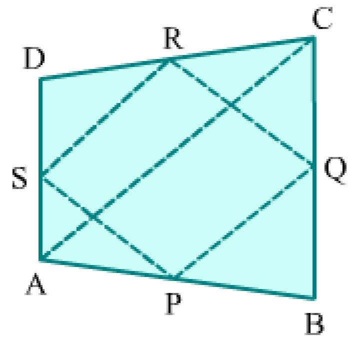
$\Rightarrow \therefore PY = \frac{1}{3}PQ$

Example 2: Prove that the quadrilateral formed by joining the midpoints of all sides of a quadrilateral is a parallelogram.

Solution:

Let ABCD be a quadrilateral and P, Q, R and S the respective midpoints of sides AB, BC, CD and DA.

Construction: Draw diagonal AC of quadrilateral ABCD.



To prove that PQRS is a parallelogram, we have to prove that it has one pair of parallel and equal opposite sides.

In ΔACD, R and S are the midpoints of sides CD and DA respectively.

So, by the midpoint theorem, we obtain SR||AC and $SR = \frac{1}{2}AC$ ... (1)

Now, in ΔABC, P and Q are the midpoints of sides AB and BC respectively.

So, by midpoint theorem, we obtain PQ||AC and $PQ = \frac{1}{2}AC$ ... (2)

From 1 and 2, we obtain:

PQ||RS(∵ Lines parallel to the same line are also parallel)

PQ = RS

In quadrilateral PQRS, we have one pair of parallel and equal opposite sides, i.e., PQ||RS and PQ = RS. Thus, PQRS is a parallelogram.